Solving Quadratics using Square Roots

- Best time (only time) to use this method:
  1. Standard Form, no bx
  2. Vertex Form

- Same concept as solving equations where the goal is to get the x by itself.

- Use the Square Root Property to help solve.

Square Root Property

- The square root of a number can be BOTH positive and negative.

  \[ \sqrt{4} = \pm 2 \]  because \( 2 \cdot 2 = 4 \) and \( -2 \cdot -2 = 4 \)

- We usually just use the positive square root, but we need to account for both when solving quadratics because this is where we get our two solutions.

- This is where we get the term "find the roots."
Find the roots.

\[ x^2 = 36 \]

Standard Form, no bx
Best Method: Square Roots
Other Methods: Graphing, Quadratic Formula, Factoring/ZPP (if Diff of Squares)

\[ \sqrt{x^2} = \pm \sqrt{36} \]

\[ x = \pm 6 \]

Solve the quadratic.

\[ x^2 - 16 = 0 \]

Standard Form, no bx
Best Method: Square Roots
Other Methods: Graphing, Quadratic Formula, Factoring/ZPP (if Diff of Squares)

\[ \pm \sqrt{16} + 16 \]

\[ x^2 = 16 \]

\[ \sqrt{x^2} = \pm \sqrt{16} \]

\[ x = \pm 4 \]
Find the x-intercepts.

\[ x^2 + 6 = 31 \]

\[
\begin{align*}
-6 & \quad -6 \\
\hline \\
x^2 & = 25 \\
\sqrt{x^2} & = \pm \sqrt{25} \\
x & = \pm 5
\end{align*}
\]

**Standard Form, no bx**  
Best Method: Square Roots  
Other Methods: Graphing, Quadratic Formula, Factoring/ZPP (if Diff of Squares)

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Find the roots.

\[ 3x^2 = 27 \]

\[
\begin{align*}
\frac{3}{3} & \quad \frac{3}{3} \\
\hline \\
x^2 & = 9 \\
\sqrt{x^2} & = \pm \sqrt{9} \\
x & = \pm 3
\end{align*}
\]

**Standard Form, no bx**  
Best Method: Square Roots  
Other Methods: Graphing, Quadratic Formula, Factoring/ZPP (if Diff of Squares)
Solve for $x$.

$$4x^2 + 1 = 325$$

$$4x^2 = 324$$

$$x^2 = 81$$

$$x = \pm 9$$

Best Method: Square Roots
Other Methods: Graphing, Quadratic Formula, Factoring/ZPP (if Diff of Squares)

Find the $x$-intercepts.

$$9x^2 + 10 = 91$$

$$9x^2 = 81$$

$$x^2 = 9$$

$$x = \pm 3$$

Best Method: Square Roots
Other Methods: Graphing, Quadratic Formula, Factoring/ZPP (if Diff of Squares)
Sometimes the radicals are not perfect squares/whole number solutions. Simplify if possible.

Solve the quadratic. Write answer in simplified radical form.

\[ x^2 = 8 \]
\[ \sqrt{x^2} = \pm \sqrt{8} \]
\[ x = \pm 2\sqrt{2} \]
Solve the quadratic.
Write the answer in simplified radical form.

\[ 10x^2 + 2 = 292 \]

\[ \sqrt{2} - \sqrt{2} \]

\[ \frac{10x^2 - 290}{10} \]

\[ x^2 = 29 \]

\[ \sqrt{x^2} = \pm \sqrt{29} \]

\[ x = \pm \sqrt{29} \]

Standard Form, no bx
Best Method: Square Roots
Other Methods: Graphing, Quadratic Formula, Factoring/ZPP (if Diff of Squares)

Find the zeros.
Write the answer in simplified radical form.

\[ 10x^2 - 10 = 470 \]

\[ \sqrt{10} \pm 10 \]

\[ \frac{10x^2 - 480}{10} \]

\[ x^2 = 48 \]

\[ \sqrt{x^2} = \pm \sqrt{48} \]

\[ x = \pm \sqrt{48} \]

\[ \sqrt{16} \sqrt{3} \]

\[ 4\sqrt{3} \]

Standard Form, no bx
Best Method: Square Roots
Other Methods: Graphing, Quadratic Formula, Factoring/ZPP (if Diff of Squares)
Find the zeros. Write the answer in simplified radical form.

**Standard Form, no bx**
Best Method: Square Roots
Other Methods: Graphing, Quadratic Formula, Factoring/ZPP (if Diff of Squares)

1. \(6x^2 - 40 = 200\)
   \[
   \frac{6x^2 - 40}{6} = \frac{200}{6}
   \]
   \(x^2 = 40\)
   \[
   \sqrt{x^2} = \pm \sqrt{40}
   \]
   \(x = \pm 2\sqrt{10}\)

2. \(16x^2 - 49 = 0\)
   \[
   \frac{16x^2 - 49}{16} = \frac{0}{16}
   \]
   \(x^2 = \frac{49}{16}\)
   \[
   \sqrt{x^2} = \pm \sqrt{\frac{49}{16}}
   \]
   \(x = \pm \frac{7}{4}\)
Find the zeros. Write the answer in simplified radical form.

\[ 36x^2 - 24 = 0 \]

**Standard Form, no bx**

*Best Method: Square Roots*

*Other Methods: Graphing, Quadratic Formula, Factoring/ZPP (if Diff of Squares)*

\[
36x^2 = 24
\]

\[
\sqrt{36} = \pm \frac{\sqrt{24}}{36}
\]

\[
x^2 = \frac{\sqrt{24}}{36}
\]

\[
x = \pm \frac{\sqrt{6}}{3}
\]

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Find the zeros. Write the answer in simplified radical form.

\[ 12x^2 - 25 = 0 \]

**Standard Form, no bx**

*Best Method: Square Roots*

*Other Methods: Graphing, Quadratic Formula, Factoring/ZPP (if Diff of Squares)*

\[
12x^2 = 25
\]

\[
\sqrt{12} = \pm \frac{\sqrt{25}}{12}
\]

\[
x^2 = \frac{25}{12}
\]

\[
x = \pm \frac{5\sqrt{3}}{6}
\]
Solving Quadratics using Square Roots
(Vertex Form)

- Same concept as solving equations where the goal is to get the x by itself.
- Use the Square Root Property to help solve.
- Remember: when taking the square root of a positive number, we must use $+$
- Vertex Form: $a(x - h)^2 + k$

Solve the quadratic.

\[
(x + 2)^2 = 0
\]

\[
\sqrt{(x+2)^2} = \pm \sqrt{0}
\]

\[
x = -2 \pm 0
\]

\[
x = -2
\]

**Vertex Form**

Best Method: Square Roots
Other Methods: Graphing

vertex is on the x-axis
Find the roots.

\[(x - 3)^2 = 4\]

\[\sqrt{(x-3)^2} = \pm \sqrt{4}\]

\[x = 3 \pm 2\]

\[x = 3 + 2 \rightarrow 3 + 2 = 5\]

\[x = 5, 1\]

**Vertex Form**

**Best Method:** Square Roots

**Other Methods:** Graphing

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Find the x-intercepts.

\[(x + 4)^2 + 9 = 0\]

\[\sqrt{(x+4)^2} = \pm \sqrt{-9}\]

**Vertex Form**

**Best Method:** Square Roots

**Other Methods:** Graphing

*no solution*
Solve the quadratic.

\[(x + 5)^2 - 6 = 10\]

\[\frac{(x+5)^2}{2} = 16\]

\[\sqrt{(x+5)^2} = \pm \sqrt{16}\]

\[x + 5 = -5 \pm 4\]

\[x = -5 \pm 4\]

\[x = -1, -9\]

---

Find the roots.

\[4(x - 8)^2 = 0\]

\[\frac{(x-8)^2}{4} = 0\]

\[\sqrt{(x-8)^2} = \pm \sqrt{0}\]

\[x - 8 = 8 \pm 0\]

\[x = 8\]
Solve the quadratic.

\[ 2(x - 6)^2 = 32 \]

\[ (x - 6)^2 = 16 \]

\[ \sqrt{(x-6)^2} = \pm \sqrt{16} \]

\[ x - 6 = 6 \pm 4 \]

\[ x = 6 \pm 4 \rightarrow 6 \pm 4 = 10, 2 \]

\[ x = 10, 2 \]

Solve the quadratic.

\[ 3(x + 7)^2 + 4 = 31 \]

\[ 3(x + 7)^2 = 27 \]

\[ 3(x + 7)^2 = \frac{27}{3} \]

\[ (x + 7)^2 = \pm \sqrt{9} \]

\[ x + 7 = 7 \pm 3 \]

\[ x = 7 \pm 3 \rightarrow 7 + 3 = 10, 7 - 3 = 4 \]

\[ x = 10, 4 \]
Sometimes the radicals are not perfect squares/whole number solutions. Simplify if possible.

Solve the quadratic. Write answer in simplified radical form.

\[(x + 3)^2 = 12\]

\[
\sqrt{(x+3)^2} = \pm \sqrt{12} \\
x \pm \sqrt{3} = \frac{-3}{2} \pm 2\sqrt{3} \\
x = -3 \pm 2\sqrt{3}
\]

Solve the quadratic. Write answer in simplified radical form.

\[(x + 2)^2 - 17 = 0\]

\[
(x + 2)^2 = 17 \\
\sqrt{(x+2)^2} = \pm \sqrt{17} \\
x \pm 2 = \frac{-2}{2} \pm \sqrt{17} \\
x = -2 \pm \sqrt{17}
\]

Vertex Form
Best Method: Square Roots
Other Methods: Graphing
Find the roots.
Write answer in simplified radical form.

\[(x - 4)^2 - 18 = 0\]
\[\frac{\pm \sqrt{18}}{\pm 18}\]
\[(x-4)^2 = 18\]
\[\sqrt{(x-4)^2} = \sqrt{18}\]
\[x-4 = \pm 3\sqrt{2}\]
\[x = 4 \pm 3\sqrt{2}\]

**Vertex Form**
**Best Method:** Square Roots
**Other Methods:** Graphing

Find the x-intercepts.
Write answer in simplified radical form.

\[3(x + 6)^2 = 135\]
\[\frac{\pm \sqrt{135}}{\pm 3}\]
\[(x+6)^2 = 45\]
\[\sqrt{(x+6)^2} = \pm \sqrt{45}\]
\[x+6 = \pm 3\sqrt{5}\]
\[x = -6 \pm 3\sqrt{5}\]

**Vertex Form**
**Best Method:** Square Roots
**Other Methods:** Graphing
Solve the quadratic. Write answer in simplified radical form.

Vertex Form
Best Method: Square Roots
Other Methods: Graphing

\[ 2(x + 7)^2 + 4 = 58 \]
\[ \frac{2(x + 7)^2}{2} = \frac{54}{2} \]
\[ (x + 7)^2 = 27 \]
\[ \sqrt{(x + 7)^2} = \sqrt{27} \]
\[ x + 7 = \pm 3\sqrt{3} \]
\[ x = -7 \pm 3\sqrt{3} \]

Solve the quadratic. Write answer in simplified radical form.

1. \[ x^2 + 7 = 88 \]
2. \[ (x - 4)^2 = 4 \]

3. \[ 8x^2 - 4 = 532 \]
4. \[ 64x^2 = 49 \]

5. \[ 4(x - 1)^2 = 32 \]
6. \[ 5x^2 - 7 = 488 \]

7. \[ -2(x + 2)^2 + 96 = 0 \]
8. \[ 4x^2 + 6 = 15 \]

9. \[ x^2 + 8 = 80 \]
10. \[ -x^2 + 37 = -12 \]
Solve the quadratic. Write answer in simplified radical form.

1. \( x^2 + 7 = 88 \)
   \[ x = \pm 9 \]

2. \( (x - 4)^2 = 4 \)
   \[ x = 2, 6 \]

3. \( 8x^2 - 4 = 532 \)
   \[ x = \pm \sqrt{67} \]

4. \( 64x^2 = 49 \)
   \[ x = \pm \frac{7}{8} \]

5. \( 4(x - 1)^2 = 32 \)
   \[ x = 1 \pm 2\sqrt{2} \]

6. \( 5x^2 - 7 = 488 \)
   \[ x = \pm 3\sqrt{11} \]

7. \( -2(x + 2)^2 + 96 = 0 \)
   \[ x = -2 \pm 4\sqrt{3} \]

8. \( 4x^2 + 6 = 15 \)
   \[ x = \pm \frac{3}{2} \]

9. \( x^2 + 8 = 80 \)
   \[ x = \pm 6\sqrt{2} \]

10. \( -x^2 + 37 = -12 \)
    \[ x = \pm 7 \]