Solving Quadratics using the Quadratic Formula

- Able to use this method to solve all quadratics that are in standard form

- Best method to use if:
  1. Standard form, $nx^2 & bx$
  2. Standard form, $x^2 & odd \ bx$

- Must memorize the quadratic formula!

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Discriminant

\[
b^2 - 4ac
\]
Using the Quadratic Formula

1. Make sure the quadratic is in standard form.

   **Standard Form:** \( ax^2 + bx + c = 0 \)

   \[
   \begin{align*}
   2x^2 + 3 &= 4x \\
   2x^2 + 3x - 5 &= 16 \\
   2x^2 + 16 &= 0 \\
   2x^2 - 4x + 3 &= 0 \\
   2x^2 + 3x - 21 &= 0 \\
   2x^2 + 0x + 16 &= 0
   \end{align*}
   \]

2. Write down values of \( a, b, \) and \( c. \)

   \[2x^2 + 3x + 5 = 0\]

   \[
   \begin{align*}
   a &= 2 \\
   b &= 3 \\
   c &= 5
   \end{align*}
   \]
3. Input the values \((a,b,c)\) into the discriminant.

***Use parenthesis when inputting numbers!***

**discriminant**: \(b^2 - 4ac\)

\[2x^2 + 3x + 5 = 0\]

\[
a = 2 \quad b^2 - 4ac \Rightarrow (3)^2 - 4(2)(5) = \boxed{31}
\]

No Solution

The discriminant tells us the number/type of solutions.

<table>
<thead>
<tr>
<th># of discriminant</th>
<th># of solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td># is positive</td>
<td>2 real solutions, no radical</td>
</tr>
<tr>
<td>(perfect square)</td>
<td></td>
</tr>
<tr>
<td># is positive</td>
<td>2 real solutions, w/ radical</td>
</tr>
<tr>
<td>(non-perfect square)</td>
<td></td>
</tr>
<tr>
<td># is zero</td>
<td>1 real solution</td>
</tr>
<tr>
<td># is negative</td>
<td>0 real solutions</td>
</tr>
<tr>
<td></td>
<td>no solution</td>
</tr>
<tr>
<td></td>
<td>(2 imaginary solutions)</td>
</tr>
</tbody>
</table>
4. Input the discriminant and the other values into the quadratic formula.

\[ x = \frac{-b \pm \sqrt{\#}}{2a} \]

5. Simplify the \(-b\), the radical, and the \(2a\).

\[ x^2 + 2x - 8 = 0 \]

\[
\begin{align*}
a &= 1 \\
b &= 2 \\
c &= -8
\end{align*}
\]

\[
\begin{align*}
b^2 - 4ac & \rightarrow (2)^2 - 4(1)(-8) = 36 \\
a &= 1 \\
b &= 2 \\
c &= -8 \\
-\frac{(2) \pm \sqrt{36}}{2(-1)} &= \frac{-2 \pm 6}{2}
\end{align*}
\]

6. Solve for your solutions.

With the \(\pm\) in front of the radical, that means we need to create two expressions, one with a plus \((+)\) and the other with a minus \((-)\).
Solve the quadratic.
\[ x^2 + 2x - 8 = 0 \]

**a = 1  \quad b = 2  \quad c = -8**

Standard Form, \( x^2/\text{even bx} \)

Best Method: PS Trinomials/Completing the Square

Other Methods: Graphing, Quadratic Formula, Factoring/ZPP (if Product Sum)

\[
\frac{-2 \pm \sqrt{36}}{2} = \frac{-2 \pm 6}{2}
\]

\[
-2 + 6 = \frac{4}{2} = 2 \quad \text{and} \quad -2 - 6 = \frac{-8}{2} = -4
\]

\[
x = 2, -4
\]

Solve the quadratic.
\[ 6x^2 - 5x - 4 = 0 \]

**a = 6  \quad b = -5  \quad c = -4**

Standard Form, \#\(x^2/bx\)

Best Method: Quad Formula

Other Methods: Graphing

\[
\frac{-(-5) \pm \sqrt{121}}{2(-6)} = \frac{5 \pm 11}{12}
\]

\[
\frac{5 + 11}{12} = \frac{16}{12} = \frac{4}{3} \quad \text{and} \quad \frac{5 - 11}{12} = \frac{-6}{12} = \frac{-1}{2}
\]

\[
x = \frac{4}{3}, -\frac{1}{2}
\]
Solve the quadratic.

4x^2 + 9x + 5 = 0

Standard Form, \#x^2/bx

Best Method: Quad Formula

Other Methods: Graphing

a = 4  \quad b^2 - 4ac \Rightarrow

b = 9  \quad (9)^2 - 4(4)(5) = 1

c = 5

\[
\frac{-9 \pm \sqrt{81 - 20}}{2(4)} = \frac{-9 \pm 1}{8} \quad \Rightarrow \quad \frac{-9+1}{8} = \frac{-8}{8} = -1
\]

\[
\frac{-9-1}{8} = \frac{-10}{8} = -\frac{5}{4} \quad \text{or}
\]

\[
\chi = -1, -\frac{5}{4}
\]

Solve the quadratics.

1) 2x^2 – 3x – 20 = 0  \quad 2) 2x^2 – 3x – 5 = 0

3) 2x^2 – x – 36 = 0  \quad 4) 5x^2 + 9x = –4

5) 4x^2 + 8x + 3 = 0  \quad 6) 5x^2 – 80 = 0
Solve the quadratics.
1) \(2x^2 - 3x - 20 = 0\)
   \[x = 4, -\frac{5}{2}\]
2) \(2x^2 - 3x - 5 = 0\)
   \[x = \frac{5}{2}, -1\]
3) \(2x^2 - x - 36 = 0\)
   \[x = \frac{9}{2}, -4\]
4) \(5x^2 + 9x = -4\)
   \[x = -\frac{4}{5}, -1\]
5) \(4x^2 + 8x + 3 = 0\)
   \[x = -\frac{1}{2}, -\frac{3}{2}\]
6) \(5x^2 - 80 = 0\)
   \[x = +4\]