Solving Quadratics by Factoring/Zero Product Property

- BEST method to use if in FACTORED FORM!
- Can use on quadratics that can be factored, BUT not all quadratics can be factored, so it limits the effectiveness of using this method except for quadratics that are in factored form.
- Two parts to solving this method:
  1. Factoring (not needed if in factored form)
  2. Zero Product Property

Factoring Review

Factoring Binomials:
1. Check for GCF
2. Check for Difference of Squares

Factoring Trinomials:
1. Check for GCF
2. Look at the $x^2$:
   - If $x^2$, then Product Sum
   - If $#x^2$, then Slide Divide
Zero Product Property

- If the product is zero, then one or more of the factors must equal zero.

$$A \cdot B = 0$$

Since we don't know if A or B is equal to zero, then solve as if both factors are equal to zero.

Solve for x.

$$(x + 2)(x - 3) = 0$$

Factored Form

Best Method: Zero Product Property
Other Methods: Graphing

$$(-2 + 2)(-2 - 3) = 0$$
$$0(-5) = 0 \checkmark$$
$$(3 + 2)(3 - 3) = 0$$
$$(5)(0) = 0 \checkmark$$

$$x = -2, 3$$
Find the x-intercepts.

\[(x - 1)(x + 7) = 0\]

Zero product property

\[
\begin{align*}
\frac{x-1}{x+7} &= 0 \\
\frac{1}{7} &= 0 \\
& \quad \frac{1}{7} + 1 \\
\frac{x}{7} &= -7 \\
x &= 1 \\
x &= -7
\end{align*}
\]

\[x = 1, -7\]

If there is no number in front of the x, then the solution will be the opposite of the number next to the x.

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Find the roots.

\[x(x - 3) = 0\]

Zero product property

\[
\begin{align*}
\frac{x}{3} &= 0 \\
\frac{1}{3} &= 0 \\
\frac{x}{3} &= 3 \\
x &= 3
\end{align*}
\]

\[x = 0, 3\]
Solve the quadratic.

\[3x(x + 2) = 0\]

**Factored Form**
**Best Method:** Zero Product Property
**Other Methods:** Graphing

\[
\begin{align*}
\text{zero product property} \\
\frac{x}{3} &= 0 \quad \frac{x + 2}{2} = 0 \\
x &= 0 \quad x = -2 \\
\end{align*}
\]

\[x = 0, -2\]

Solve for \(x\).

\[(x + 1)(3x + 2) = 0\]

**Factored Form**
**Best Method:** Zero Product Property
**Other Methods:** Graphing

\[
\begin{align*}
\text{zero product property} \\
\frac{x + 1}{1} &= 0 \quad \frac{3x + 2}{2} = 0 \\
x &= -1 \quad x = -\frac{2}{3} \\
\end{align*}
\]

\[x = -1, -\frac{2}{3}\]
Find the zeros.

\[(4x - 1)(x - 3) = 0\]

**Factored Form**

**Best Method:** Zero Product Property

**Other Methods:** Graphing

We've seen the relationship between the factors of a quadratic and the solution of a quadratic.

There is also a relationship between the factors/solutions and the x-intercepts of a quadratic when it is graphed.
The solutions to a quadratic are 1 and 4. What are the factors of the quadratic?

\[ \begin{align*}
\text{Solutions:} & \quad x = -6, -4 \\
\text{Factors:} & \quad (x + 6)(x + 4)
\end{align*} \]
If the x-intercepts are -5 and -\(\frac{1}{4}\),
What are the factors of the quadratic?

\[
\begin{align*}
X &= -5 \\
+5 &+5 \\
X + 5 &= 0 \\
(x + 5) \\
4 \cdot X &= -\frac{1}{4} \cdot 4 \\
4x &= -1 \\
+1 &+1 \\
4x + 1 &= 0 \\
(4x + 1)
\end{align*}
\]

A quadratic's roots are -\(\frac{1}{2}\) and \(\frac{3}{4}\),
What are the factors of the quadratic?

\[
\begin{align*}
2 \cdot X &= -\frac{1}{2} \cdot 2 \\
+1 &+1 \\
2x + 1 &= 0 \\
(2x + 1) \\
4 \cdot X &= \frac{3}{4} \cdot 4 \\
4x &= 3 \\
-3 &-3 \\
4x - 3 &= 0 \\
(4x - 3)
\end{align*}
\]
Solve the quadratic.

\[ x^2 + 5x + 4 = 0 \]

Trinomial > No GCF > \( x^2 \) > PS

\[ \begin{align*} &5 &1 \cdot 4 \\ & (x + 1)(x + 4) = 0 \end{align*} \]

Zero Product Property

\[ x = -1, -4 \]

Standard Form (w/ bx)

Best Method: Quad Formula
Other Methods: Graphing & Factoring/Zero Product Property

Solve for x.

\[ x^2 - 4 = 0 \]

Binomial > No GCF > DOS

\[ (x - 2)(x + 2) = 0 \]

Zero Product Property

\[ x = 2, -2 \text{ or } x = \pm 2 \]

Standard Form (w/out bx)

Best Method: Square Roots
Other Methods: Graphing, Quadratic Formula, and Factoring/Zero Product Property
Find the x-intercepts.

\[ x^2 + 8x - 9 = 0 \]

**Standard Form (w/ bx)**
Best Method: Quad Formula
Other Methods: Graphing, Factoring/Zero Product Property

**Best Method:** Quad Formula

**Other Methods:** Graphing, Factoring/Zero Product Property

Trinomial > No GCF > PS

8 \(-1\bullet 9\)

\((x - 1)(x + 9) = 0\)

Zero Product Property

\[ x = 1, -9 \]

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Solve the quadratic.

\[ 3x^2 + 7x + 2 = 0 \]

**Standard Form (w/ bx)**
Best Method: Quad Formula
Other Methods: Graphing, Factoring/Zero Product Property

**Best Method:** Quad Formula

**Other Methods:** Graphing, Factoring/Zero Product Property

Trinomial > No GCF > SD

7 \(1\bullet 6\)

\((x + 1)(x + 6) = 0\)

\(\frac{3}{3}\)

\((3x + 1)(x + 2) = 0\)

Zero Product Property

\[ x = -\frac{1}{3}, 2 \]
NOT ALL QUADRATICS ARE FACTORABLE!

That is why this method of solving quadratics is not a "best method" unless it is in factored form.

If the quadratic is in **Standard Form, w/ "bx"**...plan to use **Quadratic Formula**
If the quadratic is in **Standard Form, w/out "bx"**...plan to use **Square Roots**

1. \((x + 1)(x - 5) = 0\)  
2. \(x(x + 2) = 0\)  
3. \((4x + 5)(x + 1) = 0\)  
4. \((2x + 3)(4x - 3) = 0\)  
5. \(3x(x - 9) = 0\)  
6. \(x^2 + 8x + 15 = 0\)  
7. \(4x^2 - 6 = 3\)  
8. \(2x^2 + x - 2 = 4\)  
9. What are the factors if the solutions are 8 and \(-\frac{2}{3}\)?
1. \((x + 1)(x - 5) = 0\)
   \(x = -1, 5\)

2. \(x(x + 2) = 0\)
   \(x = 0, -2\)

3. \((4x + 5)(x + 1) = 0\)
   \(x = -\frac{5}{4}, -1\)

4. \((2x + 3)(4x - 3) = 0\)
   \(x = -\frac{3}{2}, \frac{3}{4}\)

5. \(3x(x - 9) = 0\)
   \(x = 0, 9\)

6. \(x^2 + 8x + 15 = 0\)
   \(x = -3, -5\)

7. \(4x^2 - 6 = 3\)
   \(x = \pm \frac{3}{2}\)

8. \(2x^2 + x - 2 = 4\)
   \(x = -2, \frac{3}{2}\)

9. What are the factors if the solutions are 8 and \(-\frac{2}{3}\)?
   \((x - 8)(3x + 2) = 0\)