Radicals

Components of a radical:

index  $\rightarrow$  radical

$\sqrt[\text{index}]{\text{radicand}}$

$3\sqrt[3]{8} \neq 3\sqrt[3]{8}$

$3$ is the index, indicating that it is a cube root

$3$ is the coefficient, meaning that the $3$ is being multiplied with the $\sqrt[3]{8}$
Simplified Radicals

A radical is simplified if it does not have any perfect squares as a factor in the radical.

Perfect Squares

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>$1^2 = 1$</td>
<td>$6^2 = 36$</td>
<td>$11^2 = 121$</td>
<td>$16^2 = 256$</td>
</tr>
<tr>
<td>$2^2 = 4$</td>
<td>$7^2 = 49$</td>
<td>$12^2 = 144$</td>
<td>$17^2 = 289$</td>
</tr>
<tr>
<td>$3^2 = 9$</td>
<td>$8^2 = 64$</td>
<td>$13^2 = 169$</td>
<td>$18^2 = 324$</td>
</tr>
<tr>
<td>$4^2 = 16$</td>
<td>$9^2 = 81$</td>
<td>$14^2 = 196$</td>
<td>$19^2 = 361$</td>
</tr>
<tr>
<td>$5^2 = 25$</td>
<td>$10^2 = 100$</td>
<td>$15^2 = 225$</td>
<td>$20^2 = 400$</td>
</tr>
</tbody>
</table>

Steps to Simplifying Radicals

1. Write out all of the factors of the radicand.
2. Find the factor that is the largest perfect square.
3. Write the largest perfect square factor and its matching factor, with the perfect square on the left.
4. Reduce/simplify the perfect square factor and multiply it to any coefficients. Leave the other factor under the radical.
\[2 \sqrt{98}\]
\[
\begin{array}{c}
\downarrow \\
9.98 \\
2.99 \\
9.14 \\
\end{array}
\]
\[2 \sqrt{49} \sqrt{2}\]
\[2 \cdot 7 \sqrt{2}\]
\[14 \sqrt{2}\]

\[3 \sqrt{24}\]
\[
\begin{array}{c}
\downarrow \\
1.24 \\
2.012 \\
3.038 \\
4.6 \\
\end{array}
\]
\[3 \sqrt{4} \sqrt{6}\]
\[3 \cdot 2 \cdot \sqrt{6}\]
\[6 \sqrt{6}\]
\[ \sqrt{x^{37}} = x^{18} \sqrt{x} \]

\[ \sqrt{x^{36}} \sqrt{x} \]

\[ \sqrt{x^8 y^5} = x^4 y^2 \sqrt{y} \]

\[ \sqrt{x^8} \sqrt{y^5} \]

\[ x^4 \sqrt[4]{y^5} \]

\[ y^2 \sqrt[2]{y^2} \]
\[
\sqrt{28x^7y^6} = 2x^3y^3\sqrt{7x}
\]

\[
\sqrt{120x^4yz^5} = 2x^2y^2z^2\sqrt{30xyz}
\]
Application to Solving Quadratics

\[ x^2 - 4x - 6 = 0 \]

a = 1  \quad (-4)^2 - 4(1)(-6) = 40
b = -4
\[
\frac{-(-4) \pm \sqrt{40}}{2(1)} = \frac{4 \pm 2\sqrt{10}}{2}
\]

*We will learn next lesson on how to simplify from here*
Simplifying Radicals with Variables

Remember that $x^2$, $x^4$, or any $x^{\text{even } \#}$ are all perfect squares.

\[
\sqrt{2^2} = 2 \\
\sqrt{x^2} = x \\
\sqrt{x^4} = \sqrt{x^2 \cdot x^2} = x \cdot x = x^2
\]

\[
\sqrt{X^{10}} = X^5 \\
\sqrt{X^{22}} = X^{11} \\
\sqrt{X^{34}} = X^{17}
\]
\( \sqrt{x^{\text{even #}}} \)  \( \iff \)  Simplify by dividing the exponent by 2

\[
\sqrt{x^5} = \sqrt{x^4} \cdot \sqrt{x} \\
\downarrow \quad \downarrow \\
X^2 \quad \sqrt{x}
\]
Subtract 1 from the exponent and then divide the exponent by 2. The leftover $x$ that was subtracted stays $\sqrt{x}$

\[
\sqrt{x^{11}} = x^5 \sqrt{x}
\]
\[
\sqrt{x^{25}} = x^{12} \sqrt{x}
\]
\[
\sqrt{x^{45}} = x^{22} \sqrt{x}
\]
Radicals can also be written as exponents.

\[
x^{\frac{a}{b}} = \sqrt[b]{x^a}
\]

\[
4^{\frac{1}{2}} = \sqrt[2]{4^1} = 2 \quad 8^{\frac{1}{3}} = \sqrt[3]{8^1} = 2
\]

\[
27^{\frac{2}{3}} = \sqrt[3]{27^2} = 9
\]

Simplify the radicals.

1. \( \sqrt{12} \) \quad 2. \( \sqrt{45} \) \quad 3. \( \sqrt{72} \)

4. \( \sqrt{125} \) \quad 5. \( \sqrt{162} \) \quad 6. \( 7\sqrt{80} \)

7. \( \sqrt[x^4y^5]{} \) \quad 8. \( \sqrt[18x^6y^3]{} \) \quad 9. \( \sqrt[490x^{10}y^4z^6]{} \)
Simplify the radicals.

1. $\sqrt{12}$
   $2\sqrt{3}$

2. $\sqrt{45}$
   $3\sqrt{5}$

3. $\sqrt{72}$
   $6\sqrt{2}$

4. $\sqrt{125}$
   $5\sqrt{5}$

5. $\sqrt{162}$
   $9\sqrt{2}$

6. $7\sqrt{80}$
   $28\sqrt{5}$

7. $\sqrt{x^4y^5}$
   $x^2y^2\sqrt{y}$

8. $\sqrt{18x^6y^3}$
   $3x^3y\sqrt{2y}$

9. $\sqrt{490x^{10}y^4z^6}$
   $7x^5y^2z^3\sqrt{10}$