Quadratic Word Problems

- Normally, the graph is a maximum (-x^2/opens down) because of the real life scenarios that create parabolas.

- The equation of the quadratic will be given.

- We will only be using the first quadrant because we only can use positive values.
  - (x-values is normally time)
  - (y-values is normally height)

\[ y = ax^2 + bx + c \]

\[ ax^2 \Rightarrow \text{normally } -16x^2, \text{ which represents gravity} \]
\[ bx \Rightarrow \text{represents the initial velocity } (v_0) \]
\[ c \Rightarrow \text{initial height (y-intercept)} \]
Solving Quadratics Word Problems

- 4 things that you could be asked to find:

1. Maximum X
2. Maximum Y
3. X-intercept (right)
4. Random Y
**Maximum x or x-value of the vertex**

How the question will be asked:
"How long does it take to get to its highest point/maximum point?"

\[ x = \text{time (normally)} \]

Finding the maximum x:
- Use \( \frac{-b}{2a} \) to find the x-value of the vertex

**Maximum y or y-value of the vertex**

How the question will be asked:
"How high does it get?" OR
"What is the highest point/maximum point?"

\[ y = \text{height (normally)} \]

Finding the maximum y:
- Use \( \frac{-b}{2a} \) to find the x-value of the vertex.
- Input x-value of vertex into quadratic OR type in quadratic into calculator and find in table
**X-intercept (right zero)**

How the question will be asked:
"**How long** does it take to hit/return to ____?"

Normally there are 2 solutions/zeros/x-int, but we only will use the solution on the right.

Finding the x-intercept on the right:
- Solve the quadratic (using any method).
*If answer is not a whole number, write it as a decimal (round 2 places).

**Random y-value (x-value will be given)**

How the question will be asked:
"After # seconds, **how high** is the object?"

x-value (normally time) will be given

Finding the random y-value:
- Input the given x-value into the quadratic OR
Type the quadratic in the calculator and find the corresponding y-value in the table
Jason went cliff diving in Acapulco, Mexico when vacationing with friends. His height as a function of time could be modeled by the function, \( h(t) = -16t^2 + 16t + 480 \), where \( t \) is the time in seconds and \( h \) is the height in feet.

(a) How long did it take for Jason to reach maximum height?

\[
\text{maximum } x \Rightarrow x\text{-value of the vertex} \quad x = \frac{-b}{2a} = \frac{-16}{2(-16)} = \frac{-16}{-32} = 0.5
\]

(b) What was Jason's maximum height?

\[
\text{maximum } y \Rightarrow y\text{-value of the vertex} \quad y = -16(0.5)^2 + 16(0.5) + 480 = 484 \text{ ft}
\]
Jason went cliff diving in Acapulco, Mexico when vacationing with friends. His height as a function of time could be modeled by the function, \( h(t) = -16t^2 + 16t + 480 \), where \( t \) is the time in seconds and \( h \) is the height in feet.

(c) How long did it take for him to hit the water?

\[ \text{x-intercept} \rightarrow \text{solve quadratic} \]
\[ \text{quadratic formula or graphing} \]
\[ 6 \text{ sec} \]

(d) How high above the water is he after 4 seconds?

\[ -16(4)^2 + 16(4) + 480 = \boxed{288 \text{ ft}} \]
If a toy rocket is launched from ground level with an initial velocity of 128 feet per second, then its height $h$ after $t$ seconds is given by the equation, $h(t) = -16t^2 + 128t$. How long will it take the toy rocket to get to the highest point?

\[
\text{Maximum } x \rightarrow x\text{-value of vertex } \\
\frac{-b}{2a} = -\frac{128}{2(-16)} = -\frac{128}{-32} = 4 \text{ seconds}
\]

The height ($h$) in feet of a ball $t$ seconds after being dropped is given by function, $h(t) = -16t^2 + 9$. How long will it take for the ball to get to the ground?

\[
\text{X-intercept } \rightarrow \text{ solve quadratic } \\
\text{quad formula, graphing, or solve roots } \\
\frac{-16x^2 + 9}{-16} = 0 \\
x^2 = \frac{9}{16} \\
x = \frac{3}{4} \rightarrow \frac{3}{4} = .75 \text{ seconds}
\]
Simon throws an object upward from the top of a 1200 ft tall building. The height of the object, measured in feet, t seconds after he threw it is \( h(t) = -16t^2 + 160t + 1200 \). What is the height of the object after 9 seconds?

\[
\text{Random } y \rightarrow \text{input } x \text{-value}
\]

\[
-16(9)^2 + 160(9) + 1200 = 1344 \text{ ft}
\]

The Empire State Building is 1250 feet tall. If an object is thrown upward from the top of the building at an initial velocity of 38 feet per second, its height s seconds after it is thrown is given by the function, \( h(s) = -16s^2 + 38s + 1250 \). What is the highest point that the ball will reach?

\[
\text{maximum } y \rightarrow y \text{-value of the vertex}
\]

\[
\frac{-b}{2a} = \frac{-38}{2(-16)} = \frac{-38}{-32} = 1.1875
\]

\[
-16(1.1875)^2 + 38(1.1875) + 1250 = 1272.56 \text{ ft}
\]
The Empire State Building is 1250 feet tall. If an object is thrown upward from the top of the building at an initial velocity of 38 feet per second, its height s seconds after it is thrown is given by the function, \( h(s) = -16s^2 + 38s + 1250 \). How long will it take the object to hit the ground?

\[
\text{x-intercept} \rightarrow \text{solve quadratic}
\]

\[
x = 10.106 \text{ seconds}
\]

\[
h(s) = -16s^2 + 38s + 1250
\]

\[
y_1 = 0
\]
\[
y_2 = -16x^2 + 38x + 1250
\]

\[\text{trace} \quad \text{cannot see the x-intercept on the right}\]

\[\text{zoom} \quad 0: \text{ZoomFit}\]

\[\text{window} \quad \text{change the following}:\]
\[
\text{Xmax} = 15 \quad \text{(change if you can't see right x-int)}
\]
\[
\text{Ymin} = -10
\]
\[
\text{Ymax} = 1400 \quad (+25/+50 \text{ more than what is there})
\]

\[\text{2nd} \text{trace} \quad 5: \text{Intersection} \quad \text{move to right x-in} \quad \text{enter} \quad 3x\]
A rocket is launched from atop a cliff that is 101 feet tall and has an initial velocity of 116 feet per second. The flight of the rocket can be modeled by the function, $h(t) = -16t^2 + 116t + 101$.

1. How long will it take the rocket to hit the ground?

   $x$-int $\rightarrow$ solve quadratic

   (quadratic formula or graphing)

   $a = -16 \quad b = 116 \quad c = 101$

   $b^2 - 4ac \rightarrow (116)^2 - 4(-16)(101) = 19920$

   $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow \frac{-116 \pm \sqrt{19920}}{2(-16)} \approx \frac{-116 \pm 141.138}{-32}$

   $8.036$ seconds

2. What is the maximum height of the rocket?

   $\max \ y \rightarrow y$-value of vertex

   \[ y = -16(3.625)^2 + 116(3.625) + 101 \]

   \[ y = -16(3.625)^2 + 116(3.625) + 101 \]

   $311.25$ feet

3. How long will it take the rocket to get to that height?

   $3.625$ seconds

4. How high will the rocket be after 4 seconds?

   $y = -16(4)^2 + 116(4) + 101$

   $y = 309$ ft
5. A ball is thrown upward from a height of 15 ft with an initial upward velocity of 5 ft per sec. Using \( h(t) = -16t^2 + 5t + 15 \), how long will it take the ball to hit the ground?

\[
y = \begin{cases} 
0 & \text{if } x = 0 \\
-16x^2 + 5x + 15 & \text{if } x \neq 0
\end{cases}
\]

- **X-Int** → solve the quadratic
  - Quadratic formula or graphing
  - \( y_1 = 0 \)
  - \( y_2 = -16x^2 + 5x + 15 \)
  - Trace → see x-int on right? (yes)
  - 2nd trace move cursor to right x-int, enter 3x
  - **1.137 seconds**

6. A juggler throws a ball in the air, releasing it 5 feet above the ground with an initial velocity of 15 feet per second. She catches the ball with her other hand when the ball returns to 5 feet above the ground. If the equation, \( y = -16x^2 + 15x \), gives the path of the ball from hand to hand, find how high she throws the ball.

\[
\text{max } y = y\text{-value of vertex}
\]

\[
\frac{-b}{2a} \rightarrow \frac{-15}{2(-16)} = \frac{-15}{-32} = 0.46875
\]

\[
y = -16(0.46875)^2 + 15(0.46875)
\]

\( 3.516 \text{ feet} \)
7. A trebuchet launches an object at a velocity of 35 feet per second. Using the function, \( h(t) = -16t^2 + 35t \), determine when the object will reach its highest point.

\[
\text{Max } x \rightarrow x \text{-value of vertex}
\]

\[
\frac{-b}{2a} \rightarrow \frac{-(35)}{2(-16)} = \frac{35}{32} = 1.094 \text{ seconds}
\]

8. A class launches a rocket in the air. The path of the rocket can be modeled by the function, \( y = -16x^2 + 64x + 1 \). If \( y \) is the rocket’s height (in feet) and \( x \) is time (sec). How long will it take the rocket to hit the ground?

\[ x\text{-int } \rightarrow \text{solve quadratic}
\]

\[
(\text{quadratic formula or graphing})
\]

\[ 4.016 \text{ seconds} \]
9. Mr. Painter is trying to dunk a basketball. He needs to jump 2.5 feet in the air to dunk the ball. The height that his feet are above the ground is given by the function, \( h(t) = -16t^2 + 12t \). What is the maximum height Mr. Painter’s feet will be above the ground? Will he be able to dunk the basketball?

\[
\begin{align*}
\text{max } y &= y\text{-value of vertex} \\
\frac{-b}{2a} &\to \frac{-(12)}{2(-16)} = \frac{-12}{-32} = \frac{3}{8} \\
y &= -16\left(\frac{3}{8}\right)^2 + 12\left(\frac{3}{8}\right) \\
y &= \frac{2.25 \text{ ft}}{} \\
2.25 &< 2.5
\end{align*}
\]

10. The length of a picture frame is 1 cm less than twice the width. The area is 28 cm\(^2\). Find the length and width of the photograph.

\[
\begin{align*}
x &= \text{width} \\
L \cdot W &= a \\
x(2x-1) &= 28 \\
-28 &= -28 \\
x(2x-1) &= 28 \leftarrow \text{graphing} \\
2x^2 - x - 28 &= 0 \leftarrow \text{quadratic formula} \\
x &= 4 \\
\text{Width} &= 4 \text{ cm} \\
\text{length} &= 7 \text{ cm}
\end{align*}
\]
11. When a gray kangaroo jumps, its path through the air can be modeled by $y = -0.0267x^2 + 0.8x$, where $x$ is the kangaroo’s horizontal distance traveled (in feet) and the $y$ is its corresponding height (in feet). How high can the kangaroo jump and how far can he jump?

$$\max y \text{ int}$$

$$\max y$$

$$\frac{-0.8}{2(-0.0267)} = 14.981$$

$$y = -0.0267(14.981)^2 + 0.8(14.981)$$

$$y = 5.99 \text{ ft high}$$

12. The length of a rectangle is 2 in more than three times the width. The area is 56 in$^2$. Find the perimeter of the rectangle.

$$x \quad \cdot = 56$$

$$3x + 2$$

$$2(4) + 2(3(4) + 2)$$

perimeter = 36 inches

$x(3x+2) = 56$

$-56 \ -56$

$x(3x+2) - 56 = 0 \ (\text{graph})$

$3x^2 + 2x - 56 = 0 \ (\text{quad form})$

$x = 4$
During World War I, mortars were fired from trenches 3 feet below ground level and had a velocity of 150 ft per second.

13. Write the quadratic based on the information given and known information about quadratics.

\[ y = \frac{-16}{(\text{gravity})} x^2 + 150x + -3 \]

14. Determine the highest point that the mortars reached after they had been shot.

\[ \max_y \rightarrow y\text{-value of vertex} \]

\[ 348.563 \text{ ft} \]