Parallel Lines

Parallel lines do not intersect and have the same slope, but different y-intercepts.

Example:
\[ y = 2x + 3 \]
\[ y = 2x + b \]
(b can be any number except +3)

Finding the equation of a line that is parallel to a given line and goes through a certain point.

- Since the lines are parallel, then the m (slope) will be the same as the other line's equation.

- Input the given point \((x, y)\) into \(y = mx + b\), and solve for the b-value.
What is the equation of a line that is parallel to $y = 3x + 4$ and goes through the point $(-1, -5)$?

$$y = 3x + b$$
$$-5 = 3(-1) + b$$
$$-5 = -3 + b$$
$$+3$$
$$b = -2$$

What is the equation of a line that is parallel to $y = -\frac{3}{4}x - 2$ and goes through the point $(10, -3)$?

$$y = -\frac{3}{4}x + b$$
$$-3 = -\frac{3}{4}(10) + b$$
$$-\frac{15}{2} = -15 + b$$
$$+\frac{15}{2}$$
$$\frac{9}{2} = b$$
1. What is the equation of a line that is parallel to \( y = -5x - 1 \) and goes through the point \((-8, 3)\)?

\[
y = -5x + b \quad (-8, 3) \\
3 = -5(-8) + b \\
3 = 40 + b \\
-40 = -40
\]

2. What is the equation of a line that is parallel to \( y = \frac{3}{4}x + 5 \) and goes through the point \((-4, 8)\)?

\[
y = \frac{3}{4}x + b \quad (-4, 8) \\
8 = \frac{3}{4}(-4) + b \\
8 = -3 + b \\
\frac{11}{4} = b
\]

Find the equation of the line that is parallel to the graphed line that also goes through \((1, 4)\).

\[
\begin{align*}
\text{Find equation from graph} & \quad \frac{x_1 - x_2}{y_1 - y_2} \cdot (x - x_2) = y - y_2 \\
\frac{-6 - 0}{0 - (-3)} \cdot (x - 0) &= y - 0 \\
y &= -\frac{1}{2}x - 3 \\
y &= -\frac{1}{2}x + b \quad (1, 4) \\
4 &= -\frac{1}{2}(1) + b \\
3 &= -\frac{1}{2} + b \\
\frac{9}{2} &= b
\end{align*}
\]

\[
y = -\frac{1}{2}x + \frac{9}{2}
\]
Perpendicular Lines

Perpendicular lines intersect one time at a 90° angle and have *opposite reciprocal* slopes.

They may have y-intercepts that are the same or different. If they are the same, then the two lines intersect on the y-axis. There is no impact of the y-intercept on whether it is perpendicular or not, just about the slope.

Perpendicular lines have **OPPOSITE RECIPROCAL** slopes.

**OPPOSITE RECIPROCAL**
"Flip the sign AND the slope"

\[
\begin{align*}
y &= 2x + 1 \\
y &= -\frac{3}{4}x + 6
\end{align*}
\]

\[
\begin{align*}
\downarrow & & \downarrow \\
-\frac{1}{2} & & \frac{4}{3}
\end{align*}
\]
What is the equation of a line that is perpendicular to \( y = -\frac{1}{2}x - 3 \) and goes through the point (4,6)*?

*Doesn't mean that the intersection point is at (4,6), but just that the line goes through that point.

\[
\begin{align*}
y &= 2x + b \\
6 &= 2(4) + b \\
6 &= 8 + b \\
6 - 8 &= b \\
-2 &= b
\end{align*}
\]

What is the equation of a line that goes through the point (-4, 2) and is perpendicular to the equation, \( 2x + 3y = 6 \)?

\[
\begin{align*}
8x + 3y &= 6 \quad \text{standard form} \\
-2x &\\
3y &= -\frac{2}{3}x + \frac{6}{3} \\
\frac{3y}{3} &= -\frac{2}{3}x + 2 \\
y &= -\frac{2}{3}x + 2 \\
y &= \frac{3}{2}x + b \\
2 &= \frac{3}{2}(-4) + b \\
2 &= -6 + b \\
b &= 2 + 6 \\
b &= 8 \\
y &= \frac{3}{2}x + 8
\end{align*}
\]
What is the equation of a line that goes through the point \((-8, 3)\) and is perpendicular to the equation, \(y = -4x - 1\)?

\[
y = \frac{1}{4}x + b \\
3 = \frac{1}{4}(-8) + b \\
3 = -2 + b \\
1 = b \\
\]

\[
y = \frac{1}{4}x + 5
\]

Find the equation of a line that is perpendicular to \(y = x + 3\) and goes through \((-10, -4)\).

\[
y = -x + b \\
-4 = -(-10) + b \\
-4 = 10 + b \\
-14 = b \\
\]

\[
y = -x - 14
\]
Find the equation of the line that is perpendicular to the graphed line that also goes through (-2, 3).

\[ y = -\frac{1}{2}x - 3 \]

\[ y = 2x + b \quad (-2, 3) \]

\[ 3 = 2(-2) + b \]

\[ 3 = -4 + b \]

\[ 7 = b \]

\[ y = 2x + 7 \]
Four Types of Relationships between Lines

1. Parallel
   - Slope is the same
   - Y-int is different

2. Perpendicular
   - Slope is opposite reciprocal

3. Normal Lines
   - Slope is not the same or opposite reciprocal
   - One intersection point

4. Same Lines
   - Equivalent equations (same slope, same y-intercept)
   - Infinite (∞) number of intersection points.

What type of relationship do the following lines have with the equation, \( y = \frac{1}{4} x + 3 \)?

(a) Parallel  (b) Perpendicular  (c) Normal  (d) Same

1. \( y = 4x + 3 \)  
2. \( y = \frac{1}{4} x - 3 \)
3. \( y = -\frac{1}{4} x + 3 \)  
4. \( y = -4x + 7 \)
5. \(-x + 4y = 12\)  
6. \(8x + 2y = -12\)
What type of relationship do the following lines have with the equation, $y = \frac{1}{4} x + 3$?

(a) Parallel   (b) Perpendicular   (c) Normal   (d) Same

1. $y = 4x + 3$  
   Normal

2. $y = \frac{1}{4} x - 3$  
   Parallel

3. $y = -\frac{1}{4} x + 3$  
   Normal

4. $y = -4x + 7$  
   Perpendicular

5. $-x + 4y = 12$  
   Same

6. $8x + 2y = -12$  
   Perpendicular