Factoring Trinomials

Trinomials have three terms.

Three types of trinomials to factor:
1. Trinomial with an $x^2$, like $x^2 + 4x + 3$
2. Trinomial with a $#x^2$, like $2x^2 + 7x + 3$
3. Trinomial with something else in front or more than one variable in the trinomial. If this is the case, then you are done factoring after taking the GCF.
Factor.

**Trinomial**

\[4x^2 + 6x + 10\]

- GCF:
  - 1 | 4
  - 2 | 2
  - 2 | 1
  - 1 | 0
  - GCF → 2

- \(x^2 \rightarrow SD\)
  - 2(\(ax^2 + 3x + 5\))

- Factor using slide-divide

Factor.

**Trinomial**

\[3x^4 + 9x^3 + 6x^2\]

- GCF:
  - 1 | 3
  - 3 | 3
  - 2 | 3
  - X | 1
  - GCF → 3

- \(x^2 \rightarrow x^3\)
  - \(3x^2(x^2 + 3x + 2)\)

- Factor using product-sum
Factor.

$9x^3 + 12x^2 + 15x$

Trinomial

GCF

$\# \text{ GCF} \rightarrow 3$

$X \text{ GCF} \rightarrow x$

$#x^3 \rightarrow SD$

$3x \left( 3x^2 + 4x + 5 \right)$

$\Rightarrow \text{factor using slide-divide}$
Factoring Trinomials using Product-Sum

Use this method...

1. Only if it is a trinomial,
2. AFTER checking for GCF
3. If the $x^2$ does not have a number in front of it, or has a "ghosty 1"

Factor.

\[
x^2 + 5x + 4
\]

Trinomial: \(x^2 + 5x + 4\)

GCF: 1

# GCF \(\Rightarrow 1\)

\(x\) GCF \(\Rightarrow\) 1

\(x^2\) \(\Rightarrow\) PS

\(x^2 + 5x + 4\) \(\Rightarrow\) factoring using product-sum
Need to know which is the product and which is the sum in the trinomial.

Product = Number without a variable
Sum = Number with just a single variable

\[ x^2 + 5x + 4 \]

Steps for using Product Sum

1. List out the factors of the product, including positive AND negative factors.

\[ x^2 + 5x + 4 \]
\[ \begin{array}{c|c}
1 & 4 \\
-1 & -4 \\
2 & 2 \\
-2 & -2 \\
\end{array} \]
If the product is **POSITIVE**, then the factors will BOTH be positive or BOTH be negative.

\[
\begin{array}{c}
+ 24 \\
10 - 24 \\
-10 - 24 \\
20 12 \\
-2 - 12 \\
3 8 \\
-3 - 8 \\
4 6 \\
-4 - 6 \\
\end{array}
\]

If the product is **NEGATIVE**, then one factor will be positive and the other factor will be negative.

\[
\begin{array}{c}
- 24 \\
10 - 24 \\
-10 - 24 \\
20 12 \\
-2 - 12 \\
3 8 \\
-3 - 8 \\
4 6 \\
-4 - 6 \\
\end{array}
\]
2. Find the sums of the factors and match with the trinomial's sum.

\[ x^2 + 5x + 4 \]

\[
\begin{array}{c|c|c}
5 & 1 & 4 \\
-5 & -1 & -4 \\
4 & 2 & 2 \\
-4 & -2 & -2 \\
\end{array}
\]

*We are looking for the two factors who multiply to get 4 and add to get 5.*

3. Write the matched factors in the correct form.

\[ (x+1)(x+4) \]
Factor.

Trinomial

$x^2 - 1x - 6$

GCF

\[
\begin{array}{ccc}
1 & 1 & 1.6 \\
2 & 2 & 2 \\
\end{array}
\]

\$ GCF \rightarrow 1 \$

$x$ GCF \rightarrow \emptyset

$x^2 + ps$

\[
\begin{array}{ccc}
-5 & 1 & -6 \\
5 & -1.6 & \\
-1 & 3.3 & \\
1 & -2.3 & \\
\end{array}
\]

\[
(x+2)(x-3)
\]

Factor.

Trinomial

$x^2 - 9x + 20$

GCF

\[
\begin{array}{ccc}
1 & 1 & 1.20 \\
3 & 3 & 4.5 \\
\end{array}
\]

\$ GCF \rightarrow 1 \$

$x$ GCF \rightarrow \emptyset

$x^2 + ps$

\[
\begin{array}{ccc}
2 & 1.20 & \\
-2 & -1.20 & \\
1 & 2.10 & \\
-1 & -2.10 & \\
9 & 4.5 & \\
-9 & -4.5 & \\
\end{array}
\]

\[
(x-4)(x-5)
\]
Factor.

### $x^2 + 11x + 18$

- **Trinomial**
  - $x^2 + 11x + 18$
- **GCF**
  - $\text{GCF} \rightarrow 1$
- **$x^2 \rightarrow PS$**
  - $x^2 + 11x + 18$
  - $s \quad p$
  - $19 \quad 1.18$
  - $-19 \quad -10.18$
  - $11 \quad 2.9
  - $-11 \quad -20.9$
- **Result**
  - $(x+2)(x+9)$

### $4x^2 + 8x - 12$

- **Trinomial**
  - $4x^2 + 8x - 12$
- **GCF**
  - $\text{GCF} \rightarrow 4$
- **$x^2 \rightarrow PS$**
  - $4(x^2 + 2x - 3)$
  - $s \quad p$
  - $-2 \quad 1 - 3$
  - $2 \quad -1.3$
  - **Result**
  - $4(x-1)(x+3)$
Factor.

\[2x^2 + 14x - 36\]

Helpful "Tricks" for Product-Sum Factoring

If the product is positive, then both factors are going to be the same sign of the sum.

\[x^2 + 3x + 2 \quad x^2 - 3x + 2\]

\[(x + 1)(x + 2) \quad (x - 1)(x - 2)\]

If the product is negative, then the larger factor is going to be the same sign of the sum and the smaller factor is going to be the opposite sign.

\[x^2 + 5x - 6 \quad x^2 - 5x - 6\]

\[(x - 1)(x + 6) \quad (x + 1)(x - 6)\]
Practice

1. \(x^2 - 5x - 24\)  
   \((x + 3)(x - 8)\)

2. \(x^2 - 7x + 12\)  
   \((x - 3)(x - 4)\)

3. \(3x^2 - 15x + 18\)  
   \(3(x - 2)(x - 3)\)

4. \(x^2 + 13x - 30\)  
   \((x - 2)(x + 15)\)

5. \(x^2 + 15x + 56\)  
   \((x + 7)(x + 8)\)

6. \(2x^3 + 20x^2 + 32x\)  
   \(2x(x + 2)(x + 8)\)