Factoring

- Factoring is the reverse of multiplying
- The "answer" to factoring polynomials is the multiplying polynomials "question"

Multiplying:  Factoring:
Q: 2x(6x + 3)     Q: 12x^2 + 6x
A: 12x^2 + 6x     A: 2x(6x + 3)
Factoring Process Flow Chart

- This is to give you structure to your thinking when you are given a polynomial to factor.
- If you know this chart and use it as your thought process, you will be able to successfully factor polynomials.
- Quiz on Thursday over it (fill in the chart)

**FACTORIZING**

- **Binomial**
  - Factor using GCF
  - Factor using Difference of Squares

- **Trinomial**
  - Factor using GCF
  - Look at the $x^2$, If $x^2$, then factor using Product Sum
  - If $#x^2$, then factor using Slide Divide

- **Polynomial**
  - Factor using GCF

Factors

Factors are numbers and/or variables that are multiplied together to get the product.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 \cdot 6$</td>
<td>$12$</td>
</tr>
<tr>
<td>$2 , (x + 3)$</td>
<td>$2x + 6$</td>
</tr>
<tr>
<td>$(x + 2)(x + 3)$</td>
<td>$x^2 + 5x + 6$</td>
</tr>
</tbody>
</table>

Common Factors

Common factors are factors that are shared by more than one product.

1 & 2 are common factors
Factoring using GCF

- GCF stands for "Greatest Common Factor"
- Looking for the largest factor that is shared by all the terms (2 terms in the case of binomials)
- There may be more than one common factor, but we are only looking for the biggest/largest.
- If the GCF is just 1, then there is not a GCF.
Finding Factors of a Number

1. Start with 1 and the #

2. Go up from 2 and write down factors that go evenly until they repeat.

\[
\begin{array}{c|cc}
1 & 12 \\
1 & 12 \\
2 & 6 \\
3 & 4 \\
\end{array}
\]

Find the factors.

\[
\begin{array}{c|cc}
1 & 24 \\
1 & 24 \\
2 & 12 \\
3 & 8 \\
4 & 6 \\
\end{array}
\]
Find the greatest common factor (GCF).

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>1 ( \cdot ) 16</td>
</tr>
<tr>
<td>2</td>
<td>2 ( \cdot ) 8</td>
</tr>
<tr>
<td>4</td>
<td>4 ( \cdot ) 4</td>
</tr>
</tbody>
</table>

1, 2, & 4 are common factors

4 is the GREATEST Common Factor

When finding the GCF, circle the GCF and underline the number next to the GCF.
Find the greatest common factor (GCF).

\[
\begin{array}{c}
42 \\
\underline{1 \cdot 42} \\
\underline{2 \cdot 21} \\
\underline{3 \cdot 14} \\
\underline{6 \cdot 7}
\end{array}
\quad +
\begin{array}{c}
60 \\
\underline{1 \cdot 60} \\
\underline{2 \cdot 30} \\
\underline{3 \cdot 20} \\
\underline{4 \cdot 15} \\
\underline{5 \cdot 12} \\
\underline{6 \cdot 10}
\end{array}
= 6(7 + 10)
\]

GCF = 6

Find the GCF for the following:

1) \(18 \text{ & } 27\)

\[
\begin{array}{c}
1 \cdot 18 \\
\underline{2 \cdot 9} \\
\underline{3 \cdot 6}
\end{array}
\text{ & }
\begin{array}{c}
1 \cdot 27 \\
\underline{3 \cdot 9} \\
\underline{9 \cdot 3}
\end{array}
= 9(2 + 3)
\]

GCF = 9

2) \(48 \text{ & } 64\)

\[
\begin{array}{c}
1 \cdot 48 \\
\underline{2 \cdot 24} \\
\underline{3 \cdot 16} \\
\underline{4 \cdot 12} \\
\underline{6 \cdot 8}
\end{array}
\text{ & }
\begin{array}{c}
1 \cdot 64 \\
\underline{2 \cdot 32} \\
\underline{4 \cdot 16} \\
\underline{8 \cdot 8}
\end{array}
= 16(3 + 4)
\]

GCF = 16
GCF with Variables

- Every term must have the same variable to be able to have a GCF with the variables.

<table>
<thead>
<tr>
<th>Term</th>
<th>Variable GCF Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x + 5$</td>
<td>No Variable GCF Both terms do not have an $x$.</td>
</tr>
<tr>
<td>$3x^5 + 4x^3$</td>
<td>Variable GCF Both terms have an $x$.</td>
</tr>
<tr>
<td>$3x + 5y$</td>
<td>No Variable GCF First term does not have a $y$, while the second term does not have an $x$.</td>
</tr>
<tr>
<td>$4xy + 7y$</td>
<td>Variable GCF GCF w/ $Y$ only!! the second term does not have an $x$, so there is no GCF w/ the $x$.</td>
</tr>
</tbody>
</table>

The GCF of a variable is the **smallest** exponent of the variable that is in each term.
For variables:

**Circle** the number of variables that are the GCF and **underline** the rest of the variables that are not circled.

\[ x^5 + x^3 + x^2 \]

\[ x^2 \left( x^3 + x + 1 \right) \]

Look at \( x^2 \)
\[2x^3 + x^2 + 3x\]
\[\boxed{\text{Factors}}\]
\[x \left(2x^2 + x + 3\right)\]
\[\Rightarrow \text{look at } x^2\]

\[4x^2 + 2x + 3\]
\[\boxed{\text{Factors}}\]
\[x \times x \times \emptyset \leftarrow \text{null sign}\]

\[\text{PRIME (not factorable with GCF)}\]
\[\Rightarrow \text{look at } x^2\]

\[\text{not a GCF with the x's}\]
Factoring Binomials

- Binomials have two terms.
  Examples:
  \[ x + 3 \quad 2x + 1 \]
  \[ x^2 + 4 \quad 5x^2 + 4x \]

- There are two types of factoring you must check for when factoring a binomial
  1. Factoring using GCF
  2. Factoring using Difference of Squares
Factor.

$$4x + 8$$

\[ \begin{array}{cc}
\hline
4 & 1.8 \\
2 & 20 \\
\hline
\end{array} \]

\[ x \quad q \]

$$4(x + 2)$$

*Check for DOS*

---

Factor.

$$8x^4 + 3x^2$$

\[ \begin{array}{cc}
\hline
8 & 3 \\
2^4 & \\
XXXX & XX \\
\hline
\end{array} \]

$$x^2(8x^2 + 3)$$

*Check for DOS*
Factor.

1. $32x + 8$
   - $1032 \div 16 = 68$
   - $2 \div 16 = 2.4$
   - $4 \div 8 = 0$
   - $8(4x + 1)$
   - $\Rightarrow$ check for OOS

2. $10x^2 - 8x$
   - $1 \div 10 = 1.8$
   - $2 \div 5 = 0.4$
   - $x \div x = x$
   - $2x(5x + 4)$
   - $\Rightarrow$ check for OOS

Factor.

$18x^3 + 33x^2y$

- $1 \div 18 = 1.33$
- $2 \div 9 = 0.33$
- $3 \div 6 = 0.11$
- $\Rightarrow$ check for OOS

- $3x^2(6x + 11y)$
- $\Rightarrow$ check for OOS
Homework

Factor.

1. $9x^4 + 6x^3$  
   $3x^3(3x + 2)$

2. $4x^2y + 10xy^4$  
   $2xy(2x + 5y^3)$

3. $6x^3 - 36x$  
   $6x(x^2 - 6)$

4. $40x^2y^2 + 20xyz$  
   $20xy(2xy + z)$

BELL RINGER

Factor the following:

1. $15x^2 - 18x$  
   $3x(5x - 6)$

2. $48x^2y^3 + 72xy^5$  
   $24xy^3(2x + 3y^2)$
**Factoring Binomials using Difference of Squares**

Let's break down the name:

Difference ➔ Answer to a subtraction problem

Squares ➔ Perfect squares
   - Products of numbers that are multiplied by themselves
   
   \[ 2 \cdot 2 = 2^2 = 4 \] ➔ perfect square
   
   \[ 6 \cdot 6 = 6^2 = 36 \] ➔ perfect square
Must be able to recognize a perfect square in order to factor using difference of squares.

There will be a quiz on Thursday over all of the perfect squares from 1 to 400.

Remember:
Perfect squares are just products of numbers that are multiplied by themselves!

<p>| | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^2$</td>
<td>$= 1$</td>
<td>$6^2$</td>
<td>$= 36$</td>
</tr>
<tr>
<td>$2^2$</td>
<td>$= 4$</td>
<td>$7^2$</td>
<td>$= 49$</td>
</tr>
<tr>
<td>$3^2$</td>
<td>$= 9$</td>
<td>$8^2$</td>
<td>$= 64$</td>
</tr>
<tr>
<td>$4^2$</td>
<td>$= 16$</td>
<td>$9^2$</td>
<td>$= 81$</td>
</tr>
<tr>
<td>$5^2$</td>
<td>$= 25$</td>
<td>$10^2$</td>
<td>$= 100$</td>
</tr>
</tbody>
</table>
Difference of Squares is a special type of factoring ONLY for binomials.

All must be true in order to factor using Difference of Squares.
1. Operation between terms must be subtraction.
2. All numbers and variables must be perfect squares.

Perfect Squares with Variables

\[ \sqrt{4} = 2 \cdot 2 \quad \sqrt{16} = 4 \cdot 4 \]

so that means...

\[ \sqrt{x^2} = x \cdot x \quad \sqrt{x^4} = x^2 \cdot x^2 \]

- \(x^2\) is a perfect square
- \(x^4\) is a perfect square
- \(x^n\) is a perfect square if the \(n\) is even
Factor.

\[ x^2 - 16 \]

\[ \begin{array}{c|c}
1 & 1 \\
-1 & 1.16 \\
X & 2.8 \\
X & 4.4 \\
\hline
Q & \\
\end{array} \]

no GCF \rightarrow check for DoS

\[ (x - 4)(x + 4) \]

\[ x^2 + 4x - 4x - 16 \]

Factor.

\[ 9x^2 - 25 \]

\[ \begin{array}{c|c}
1.9 & 1.25 \\
3.3 & 5.5 \\
\hline
x & 0 \\
\end{array} \]

no GCF \rightarrow check for DoS

\[ (3x - 5)(3x + 5) \]
Factor.

\[ 4x^2 - 1 \]

\[
\begin{array}{c|c}
4 & 1 \\
\hline
4 & 1 \\
\end{array}
\]

\[ x \times 1 \]

\[ 4 \times \frac{4}{4} \]

\[ \checkmark \]

no GCF \( \Rightarrow \) check for D0S

\[ (2x-1)(2x+1) \]

Factor.

\[ 4x^2 - 16 \]

\[
\begin{array}{c|c}
4 & 16 \\
\hline
4 & 16 \\
\end{array}
\]

\[ x \times 4 \]

\[ 4 \times \frac{4}{4} \]

\[ \checkmark \]

check for D0S

\[ 4(x^2 - 4) \]

\[ 4(x - 2)(x + 2) \]
Factor.

225x^2 - 25

\[
\begin{array}{c}
9 \ 25 \\
x \cdot x
\end{array}
\]

\[25(9x^2 - 1)\]

Check for DoS

\[25(3x-1)(3x+1)\]

Factor.

64x^3 - 16x

\[
\begin{array}{c}
4 \ 16 \\
\times \times \times
\end{array}
\]

\[16x(4x^2 - 1)\]

Check for DoS

\[16x(2x-1)(2x+1)\]
Factor.

$9x^4 - 81x^2$

\[
\begin{array}{c|c}
9 & 99 \\
\hline
9x & XX \\
\end{array}
\]

$9x^2(x^2 - 9) \checkmark$

check for $D_0 S$

$9x^2(x-3)(x+3)$

Factor.

$4x^2 - 9y^2$

\[
\begin{array}{c|c}
1 & 9 \\
\hline
4 & 09 \\
\hline
2 & 3.3 \\
\hline
x & 0 \\
\hline
y & \emptyset \\
\end{array}
\]

no GCF → check for $D_0 S$

$(2x - 3y)(2x+3y)$
Factoring Possibilities w/ Binomials

When factoring a binomial, there are only 3 possibilities:

1. GCF only
   [Binomial is being added/not perfect square.]

2. No GCF, factor using Difference of Squares

3. Factor using GCF and Difference of Squares