Absolute Value

The distance (on a number line) between a number and its origin (normally 0).

\[ |3| = 3 \quad |-3| = 3 \]

1-Variable Absolute Value Equations

\[ |x + 2| = 4 \]

The expression "x + 2" is equal to 4 or -4 because the absolute value of both is 4.

\[ |2 + 2| = |4| = 4 \]

\[ |-6 + 2| = |-4| = 4 \]
An absolute value expression, by itself, CANNOT be equal to a negative because the absolute value of a number/expression is always positive!

\[ |x| = -2 \quad |3x + 1| = -4 \quad |x + 4| = -3 \]

NO SOLUTION

**Solving 1-Variable Absolute Value Equations**

1. Get the absolute value expression (what's inside the sticks) by itself using inverse order of operations.

2. Check and see if the absolute value expression is equal to a negative value (if so, there is no solution).

3. Write two equations to represent the two possible values of the absolute value expression (one with a positive solution, one with a negative solution)

4. Solve both equations for the possible values of x that make the equation true.
Solve the equation.

\[ |x + 3| = 5 \]

\[ x + \frac{3}{6} = \frac{5}{6} \]

\[ x = \frac{2}{3} \]

\[ x = -\frac{8}{3} \]

\[ x = 2, -8 \]

Solve for \( x \).

\[ |3x - 4| = 5 \]

\[ 3x - \frac{4}{6} = \frac{5}{6} \]

\[ 3x = \frac{9}{3} \]

\[ x = 3 \]

\[ x = \frac{-1}{3} \]

\[ x = 3, -\frac{1}{3} \]
Solve for $x$.

$|x + 3| + 2 = 5$

\[ x + 3 = 3 \quad \text{or} \quad x + 3 = -3 \]

\[ x = 0 \quad \text{or} \quad x = -6 \]

$x = 0, -6$

Solve for $x$.

$|x + 8| - 5 = 2$

\[ x + 8 = 7 \quad \text{or} \quad x + 8 = -7 \]

\[ x = -1 \quad \text{or} \quad x = -15 \]

$x = -1, -15$
Solve for x.

\[ 4|x + 8| = 56 \]

\[ \frac{4}{4} \]

\[ |x + 8| = 14 \]

\[ x + 8 = 14 \]
\[ x + 8 = -14 \]

\[ \frac{8}{8} \]
\[ \frac{-8}{8} \]

\[ x = 6 \]
\[ x = -22 \]

\[ x = 6, -22 \]

A cannot distribute with an absolute value

Solve for x.

\[ |x - 5| = 5 \cdot 8 \]

\[ \frac{8}{8} \]

\[ |x - 5| = 40 \]

\[ x - 5 = 40 \]
\[ x - 5 = -40 \]

\[ \frac{5}{5} \]
\[ \frac{-5}{5} \]

\[ x = 45 \]
\[ x = -35 \]

\[ x = 45, -35 \]
Solve for $x$.  

\[-10|x + 2| = -70\]

\[\frac{-10}{-10} \quad \frac{-70}{-10} \]

\[|x + 2| = 7\]

\[x + 2 = \frac{7}{2} \quad x + 2 = -\frac{7}{2}\]

\[x = 5 \quad x = -9\]

$x = 5, -9$

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Solve the equation.  

\[-4|x - 2| - 9 = -37\]

\[\frac{-4}{-4} \quad \frac{-9}{-4}\]

\[-4|x - 2| = -28\]

\[|x - 2| = 7\]

\[x - 2 = \frac{7}{2} \quad x - 2 = -\frac{7}{2}\]

\[x = \frac{9}{2} \quad x = \frac{1}{2}\]

$x = 9, -5$
Solve the equation.

\[ 3 - 1|x + 3| = 6 \]

\[ \frac{-3}{-1} \]

\[ \frac{|x + 3| = 3}{-1} \]

\[ |x + 3| = -3 \]

No solution

Solve the equation.

\[ 4 - 9|-x - 6| = -14 \]

\[ \frac{-4}{-9} \]

\[ \frac{-9|-x - 6| = -18}{-9} \]

\[ |-x - 6| = 2 \]

\[ \begin{align*}
  -x - 6 &= 2 \\
  \frac{-x}{-1} &\quad \frac{6}{-1} \\
  x &= 8 \\
  x &= -8 \\
  x &= -8, -4
\end{align*} \]
Avery is solving the following equation.

\[-2 |m - 2| = -6\]

Which equation is equivalent to Avery's equation?

A. \(-2m + 4 = -6\)  
B. \(|m - 2| = -3\)  
C. \(2m + 4 = 6\)  
D. \(|m - 2| = 3\)

Hayden solved this equation using steps shown.

\[|x - 3| = 4\]

Step 1: \(x - 3 = 4\)  
Step 2: \(x - 3 + 3 = 4 + 3\)  
Solution: \(x = 7\)

Based on the definition of absolute value, what additional step should Hayden do to determine the complete solution to the equation?

A. Hayden should include the opposite of 7 as a second solution.  
B. Hayden should include the reciprocal of 7 as a second solution.  
C. Hayden should also solve for \(-x + 3 = -4\) to determine a second solution.  
D. Hayden should also solve for \(x - 3 = -4\) to determine a second solution.
Absolute value functions are a type of **piecewise function** because it can be written in 2 pieces.

\[ y = |x| \]

\[ f(x) = \begin{cases} 
  x, & \text{if } x \geq 0 \\
  -x, & \text{if } x < 0 
\end{cases} \]
Piecewise Functions

- Functions that behave differently based on the input (domain/x) value.

- They will have more than one "piece" and can include more than one function type.

- Need to be able to find the value of the output from a graph and from a function (equation).

****REMEMBER***

An ordered pair is included in the solution set if it has a shaded circle.

An ordered pair is **NOT** included in the solution set if it has an open/non-shaded circle.
Finding an Output Value from a Piecewise Graph

f(3) means that "when 3 is input into function f", so look what the output value is when x = 3.

\[ f(-1) = 5 \]

Find:

\[ f(1) = \_\_ \]
\[ f(-4) = -2 \]
\[ f(-2) = 6 \]
\[ f(0) = 0 \]
\[ f(-3) = -2 \]
Finding an Output Value from a Piecewise Function

The "function" looks complicated, but read it from **right to left**, starting with the domain.

\[
f(x) = \begin{cases} 
-2x - 4 & \text{if } x \leq 2 \\ 
4x - 9 & \text{if } x > 2 
\end{cases}
\]

"If the domain (x-value) is less than or equal to 2, then use the function, -2x - 4."

"If the domain is greater than 2, then use the function, 4x - 9."

Find:

\[
\begin{align*}
  f(4) &= 0 \\
  f(-2) &= 4 \\
  f(2) &= 2 \\
  f(3) &= 2 \\
  f(0) &= 2
\end{align*}
\]
\[ f(x) = \begin{cases} 
-x - 4 & , \quad x < 3 \quad \times \\
x^2 - 7 & , \quad 3 \leq x \leq 10 \quad \checkmark \\
\frac{120}{x} + 5 & , \quad x > 10 \quad \times 
\end{cases} \]

Evaluate the function at the specified input.

\[ f(4) = \boxed{9} \quad (4)^2 - 7 \]

\[ f(x) = \begin{cases} 
2x + 1 & \text{if } x < 1 \\
-2x + 3 & \text{if } x \geq 1 
\end{cases} \]

\[
\begin{align*}
\text{f(4) &= } -2(4) + 3 \\
&= -5 \\
\text{f(-3) &= } 2(-3) + 1 \\
&= -5 \\
\text{f(1) &= } -2(1) + 3 \\
&= 1
\end{align*}
\]
\[ f(x) = \begin{cases} 
  x - 1 & \text{if } -x \leq -2 \\
  2x - 1 & \text{if } -2 < -x \leq 4 \\
  -3x + 8 & \text{if } -x > 4 
\end{cases} \]

\[ f(-2) = x - 1 \]
\[ = (-2) - 1 \]
\[ = -3 \]

\[ f(x) = \begin{cases} 
  x + 5 & \text{if } -x < -2 \\
  -4 & \text{if } -x \geq -2 
\end{cases} \]

\[ f(-2) = -4 \]
Based on the table shown below, which statement is correct?

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
</table>
| $f(3)$ where $f(x) = \begin{cases} 
2x - 7, & \text{if } x \leq 1 \\
-x + 9, & \text{if } x > 1 
\end{cases}$ | $f(2)$ where $f(x) = \begin{cases} 
x + 2, & \text{if } x < 8 \\
3x - 3, & \text{if } x \geq 8 
\end{cases}$ |

$\begin{align*}
-(3) + 9 &= 6 \\
(2) + 2 &= 4
\end{align*}$

(a) The output in Column A is greater.
(b) The output in Column B is greater.
(c) The two quantities are equal.
(d) The relationship cannot be determined.