Expression - number and/or variables that have a value, whether known or unknown

Equation - two expressions that have the same value (or are equal to each other in value)

Solving an Equation - the process to find a missing value that is represented by a variable(s) in an equation using inverse order of operations

***IMPORTANT***

When we solve equations, understand that all we are really doing is finding the input for the function rule when the output is given.

(Think the reverse of evaluating functions.)

\[
\begin{align*}
f(x) &= 2x + 1 \\
f(2) &= \, ? \\
Solving \text{ for } y. \\
5 &= 2x + 1 \\
x &= \, ? \\
Solving \text{ for } x.
\end{align*}
\]
The expression '2x + 3' has the same (equal) value as the expression '7'.

In order to find the value that is represented by 'x', we have to get the variable on one side of the equation by itself.

\[ x = \square \]

Order of Operations

P - Parenthesis or grouping symbols...( ), [ ], { }
E - Exponents, including roots
M/D - Multiplication and Division from L to R
A/S - Addition and Subtraction from L to R

***REMEMBER***

When we solve equations, we use the INVERSE or backwards order of this!!!
Exponents & Roots

Exponents  \( \Rightarrow \) base^ exponent

Roots  \( \Rightarrow \) \( ^\sqrt{} \) (square root)
\( ^\sqrt[3]{} \) (cube root)

Fractional Exponent  \( \Rightarrow \) base^n/d

- numerator = exponent (power) of base
- denominator = index (#) of the root

\[ 9^{1/2} = \sqrt{9^1} \quad 8^{1/3} = \sqrt[3]{8^1} \]
\[ 3x^{3/4} = \sqrt[4]{(3x)^3} \]

Multiplication & Division

Division is just multiplication of the reciprocal.

If you are having to divide by a fraction, it is easier to multiply by the reciprocal rather than dividing.

\[ 8 \div \frac{2}{1} = 4 \quad 8 \times \frac{1}{2} = 4 \]

Addition & Subtraction

Subtraction is just addition with negatives.

\[ 5 - 2 = 3 \quad 5 + (-2) = 3 \]
\[
3 + 2x = 7
\]

1. Subtract 3
2. Divide by 2

\[
\begin{align*}
\frac{-3}{a} & \quad -3 \\
\frac{2x}{2} & = \frac{4}{2} \\
\hline
\end{align*}
\]

\[
X = 2
\]

\[
\frac{3}{4}x - 2 = 4
\]

\[
\begin{align*}
\frac{-2}{0} & \quad +2 \\
\frac{4}{3} \cdot \frac{3}{4} = \frac{12}{12} & = 1 \\
\hline
\end{align*}
\]

\[
\begin{align*}
\frac{3}{4}x & = \frac{6 \cdot 4}{3} = \frac{24}{3} \\
\hline
X & = 8
\end{align*}
\]
There are certain situations where you must do something to get the equation "solvable" before using the inverse order of operations.

1. Variables on both sides of the equation
2. Distributive property
3. Fraction "covering" the variable

\[ 4 + 5x = -3x - 20 \]

\[ \begin{align*}
+3x & \quad +3x \\
\hline
4 & \quad 0
\end{align*} \]

\[ \begin{align*}
+8x & = -20 \\
-4 & \\
\hline
0 & \\
8x & = -24 \\
\hline
8 & \\
\frac{8x}{8} & = \frac{-24}{8} \\
\hline
x & = -3
\end{align*} \]
$4x + 3 = 6(2x + 1)$

$4x + 3 = 12x + 6$

$\underline{-12x}$

$-8x + 3 = 6$

$\underline{-3}$

$-8x = 3$

$x = -\frac{3}{8}$

\[
\frac{3x + 5}{4} = 2
\]

What this actually means is...

\[
\frac{3}{4}x + \frac{5}{4} = 2
\]
\[
\frac{3x + 5}{4} = 2.
\]

\[
3x + 5 = 8
\]

\[
\frac{3x}{3} = \frac{3}{3}
\]

\[
x = 1
\]

1. \(24 - 6x = 6(4 - 10)\)  
2. \(-7 + 11x = 9 - 5x\)

3. \(10(-4 + x) = 2x\)  
4. \((60 + 16x) = 12 + 4x\)

5. \(\frac{3}{4}(24 - 8x) = 2(5x + 1)\)  
6. \(8x - 4(-5x - 2) = 12x\)

7. \(\frac{1}{2}(12x - 4) = 14 - 10x\)  
8. \(-2(6 - 10x) = 10(2x - 6)\)
1. $24 - 6x = 6(4 - 10)$  
   $x = 10$  

2. $-7 + 11x = 9 - 5x$  
   $x = 1$  

3. $10 (-4 + x) = 2x$  
   $x = 5$  

4. $(60 + 16x) = 12 + 4x$  
   $x = -4$  

5. $\frac{3}{4} (24 - 8x) = 2 (5x + 1)$  
   $x = 1$  

6. $8x - 4(-5x - 2) = 12x$  
   $x = -\frac{1}{2}$  

7. $\frac{1}{2} (12x - 4) = 14 - 10x$  
   $x = 1$  

8. $-2(6 - 10x) = 10 (2x - 6)$  
   $x = \text{no solution}$